1. Show that \( \{ |+\rangle := \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right), |−\rangle := \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right) \} \) forms an orthonormal basis for \( \mathbb{C}^2 \) (i.e. show that the two vectors are orthogonal and that they have norm 1).

2. In this exercise you will show how to convert a state from the computational basis \( \{ |0\rangle, |1\rangle \} \) to the Hadamard basis \( \{ |+\rangle, |−\rangle \} \). 

   (a) Write the vector \( |0\rangle \) in the Hadamard basis (i.e. find the values \( \alpha \) and \( \beta \) such that \( |0\rangle = \alpha |+\rangle + \beta |−\rangle \)). Do the same thing for \( |1\rangle \).

   (b) Let \( |\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle \). Write \( |\psi\rangle \) in the Hadamard basis.

3. Let \( |\psi\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{3}} |01\rangle + \frac{1}{\sqrt{6}} |11\rangle \), \( |\phi\rangle = |10\rangle \), \( |\rho\rangle = \frac{1}{\sqrt{2}} (|00\rangle - i |11\rangle) \) and \( |\gamma\rangle = \frac{1}{\sqrt{2}} (i |00\rangle + |11\rangle) \). Compute

   (a) \( \langle \psi | \psi \rangle \), \( \langle \phi | \rho \rangle \), \( \langle \gamma | \gamma \rangle \)

   (b) \( \langle \psi | \phi \rangle \), \( \langle \psi | \rho \rangle \), \( \langle \gamma | \rho \rangle \)

   (c) \( \langle \psi | \rho \rangle \), \( \langle \rho | \rho \rangle \), \( \langle \gamma | \gamma \rangle \)

4. Show that for every quantum state \( |\psi\rangle \):

   (a) \( \langle \psi | \psi \rangle = 1 \);

   (b) \( |\psi\rangle \langle \psi | \) is a projector (\( P \) is a projector if \( P^2 = P \)).

5. Find the matrix that represents the unitaries

   \[
   Z |b\rangle = (-1)^b |b\rangle \\
   P |b\rangle = i^b |b\rangle \\
   c(X) |a\rangle |b\rangle = |a\rangle |a \oplus b\rangle
   \]

   where \( b \in \{0, 1\} \) and \( \oplus \) denotes the XOR operation.

6. Show that if \( U_1 \) and \( U_2 \) are unitaries then \( U_1 U_2, U_1 \otimes U_2 \) and \( c(U_1) \) are unitaries.

7. Let \( |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \). Compute \( H |\psi\rangle \) (i.e. find the values of \( \alpha' \) and \( \beta' \) such that \( H |\psi\rangle = \alpha' |0\rangle + \beta' |1\rangle \)).

8. Find the outcomes and the corresponding probabilities and post-measurement states of the measurement of both qubits of the state \( \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \) in the computational basis. Do the same for the measurement of both qubits in the Hadamard basis.

9. Let \( |\psi\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{6}} |11\rangle \). We measured the second qubit of \( |\psi\rangle \) in the computational basis and the outcome was 1. Then, we measured the first qubit (of the measurement state after the first measurement) in the Hadamard basis. Compute the outcomes and corresponding probabilities and post-measurement states of the second measurement.